Measurement Uncertainty

How are uncertainties written?

- Sometimes people use “error” and “uncertainty” interchangeably
  - Uncertainty represents the resolution/precision of your measuring device(s)
- Standard way to write a measurement and its corresponding uncertainty:

Measurement ± (Absolute)Uncertainty Units

Examples:

37.5 ± 0.5 g

127 ± 1 mm

78.3 ± 1.2 cm^3

Types of Uncertainties:

1. Absolute
2. Relative or Fractional
3. Percent
4. Min-Max

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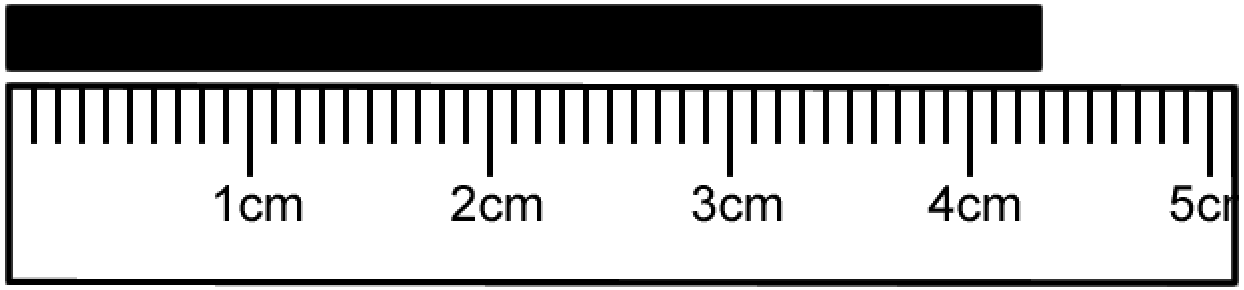
3. Percent
4. Min-Max

Absolute Uncertainty (analogue)

- Absolute Uncertainty tells you about the resolution of your measuring device and always has the *same units as the measuring device
- How to Calculate: **Absolute Uncertainty= ½ * smallest increment on measuring device**

- Example 1: How long is the black box in mm with uncertainty?
  -



  - *Units can be converted
    Example: 1mm = 0.001m

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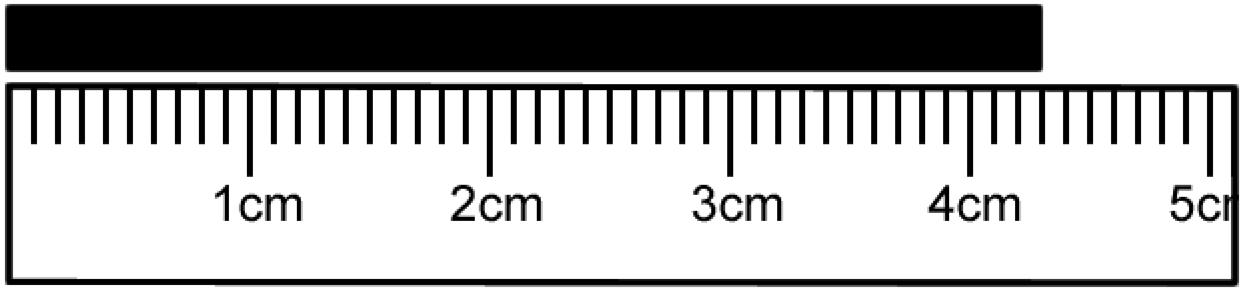
  - 43 ± 0.5 mm



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    Example: 1mm = 0.001m

Absolute Uncertainty (analogue)

- What is the absolute uncertainty of this measuring tape in inches?

Absolute Uncertainty (analogue)

- What is the absolute uncertainty of this measuring tape in inches?
- 1/16 in

Absolute Uncertainty (digital)

- What is the absolute uncertainty of this balance?

Absolute Uncertainty (digital)

- What is the absolute uncertainty of this balance?
- 0.01 g

Propagating Absolute Uncertainty

- If you were to weigh an empty vessel then weigh the vessel with a sample, what would the absolute uncertainty of the sample be if the mass of empty vessel and the mass vessel with the sample were differenced ($mass_{vessel+sample} – mass_{empty\ vessel} = mass_{sample}$) to find the mass of the sample?

Propagating Absolute Uncertainty

- If you were to weigh an empty vessel then weigh the vessel with a sample, what would the absolute uncertainty of the sample be if the mass of empty vessel and the mass vessel with the sample were differenced ($mass_{vessel+sample}$ – $mass_{empty\ vessel}$ = $mass_{sample}$) to find the mass of the sample?
- 0.02 g

NEVER SUBTRACT UNCERTAINTIES FROM ONE ANOTHER

You add the absolute uncertainties of the $mass_{vessel+sample}$ (0.01g) and $mass_{empty\ vessel}$ (0.01g) to get 0.02g as the absolute uncertainty of the $mass_{sample}$

Types of Uncertainties:

1. Absolute
2. **Relative or Fractional**
3. Percent
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Relative Uncertainty

- How to calculate from standard form: Measurement ± Absolute Uncertainty

$$\text{Relative Uncertainty} = \frac{\text{Absolute Uncertainty}}{\text{Measurement}}$$

- **Example 1:** What is the relative uncertainty of one night stand with a length of 73.2 cm if you are using a ruler that measures mm?

Relative Uncertainty

- How to calculate from standard form: Measurement ± Absolute Uncertainty

$$\text{Relative Uncertainty} = \frac{\text{Absolute Uncertainty}}{\text{Measurement}}$$

- **Example 1:** What is the relative uncertainty of one night stand with a length of 73.2 cm if you are using a ruler that measures mm?
  - Step 1 : Find Absolute Uncertainty
  - ½ * 1mm = 0.5 mm= absolute uncertainty
  -
  -

Relative Uncertainty

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- **Example 1:** What is the relative uncertainty of one night stand with a length of 73.2 cm if you are using a ruler that measures mm?
  - Step 1 : Find Absolute Uncertainty

  ½ * 1mm = 0.5 mm= absolute uncertainty
  - Step 2 convert uncertainty to same units as measurement (cm): $x = 0.05$ cm

$$\frac{1\text{ cm}}{10\text{ mm}} = \frac{x\text{ cm}}{0.5\text{ mm}}$$

Relative Uncertainty

- How to calculate from standard form: Measurement ± Absolute Uncertainty

$$\text{Relative Uncertainty} = \frac{\text{Absolute Uncertainty}}{\text{Measurement}}$$

- **Example 1:** What is the relative uncertainty of one night stand with a length of 73.2 cm if you are using a ruler that measures mm? 0.00007
  - Step 1 : Find Absolute Uncertainty

½ * 1mm = 0.5 mm= absolute uncertainty

  - Step 2 convert to cm: $x = 0.05$ cm

$$\frac{1\text{ cm}}{10\text{ mm}} = \frac{x\text{ cm}}{0.5\text{ mm}}$$

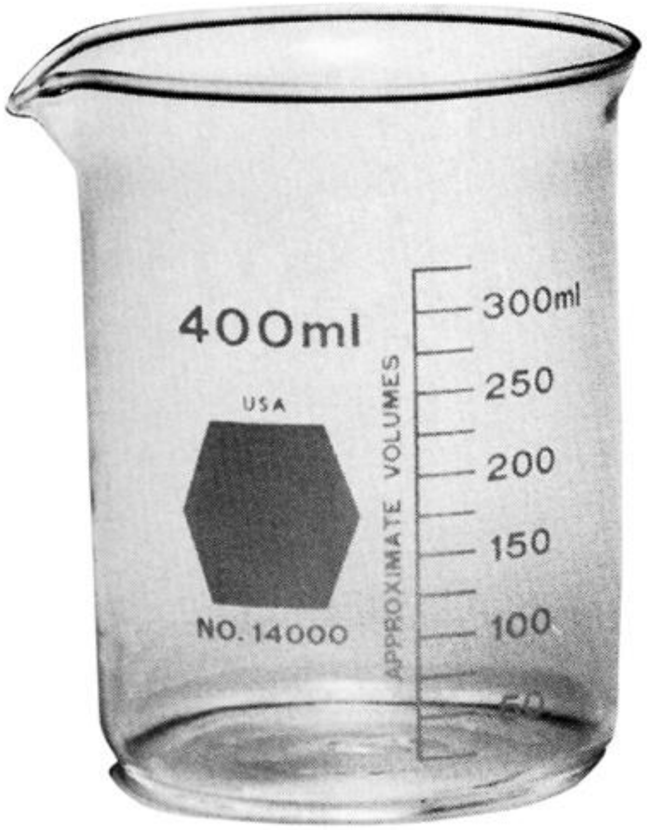
  - Step 3: Calculate Relative Uncertainty

$$\frac{0.05\text{ cm}}{73.2\text{ cm}} = 0.00068306\ldots = 0.0007$$

If uncertainty precision is smaller than precision of measuring device (exceeds significant figures), round to one decimal place

Relative Uncertainty

- The beaker on the right has a measurement increment of 25 ml
- **Example 2:** What is the relative uncertainty of 100 ml measured in this beaker?



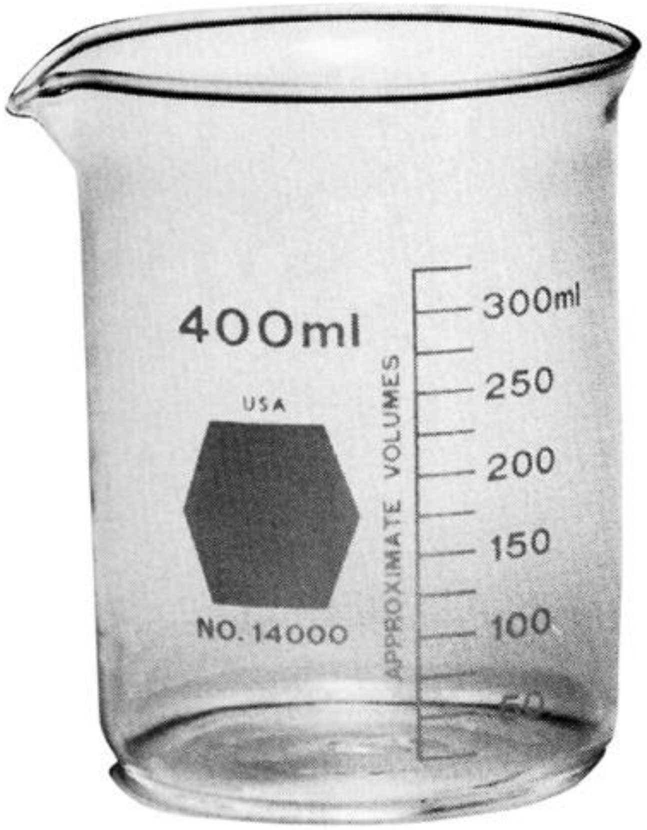
- **Example 3:** What is the relative uncertainty of 300 ml?

Relative Uncertainty

- The beaker on the right has a measurement increment of 25 ml
- **Example 2:** What is the relative uncertainty of 100 ml measured in this beaker?

1. Find absolute uncertainty of the beaker

½ * 25 ml = 12.5 ml = absolute uncertainty



- **Example 3:** What is the relative uncertainty of 300 ml?

Relative Uncertainty

- The beaker on the right has a measurement increment of 25 ml
- **Example 2:** What is the relative uncertainty of 100 ml measured in this beaker?

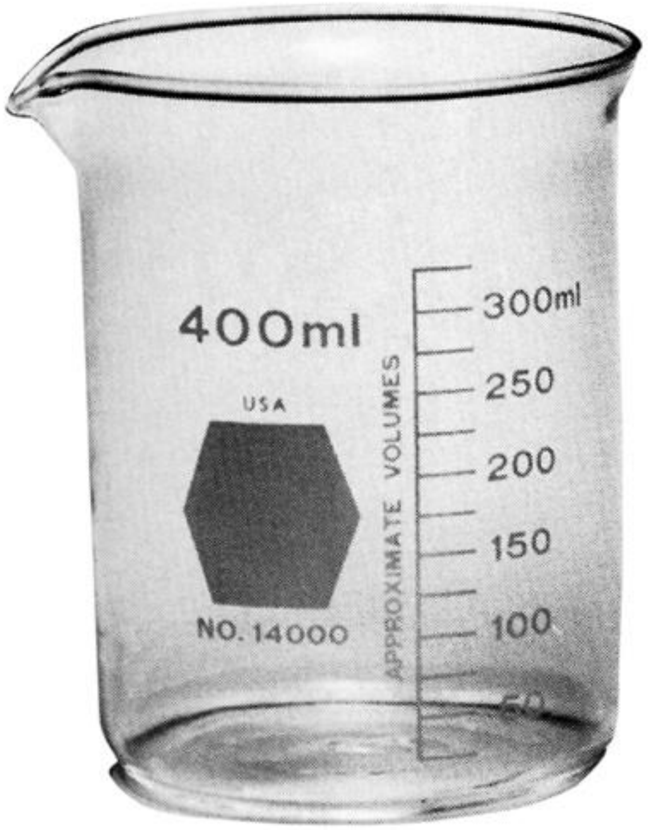
1. Find absolute uncertainty of the beaker

$\frac{1}{2}$ * 25 ml = 12.5 ml = absolute uncertainty

2. Calculate relative uncertainty

$$\frac{12.5\ \text{ml}}{100\ \text{ml}} = 0.125$$

- **Example 3:** What is the relative uncertainty of 300 ml?



Relative Uncertainty

- The beaker on the right has a measurement increment of 25 ml
- **Example 2:** What is the relative uncertainty of 100 ml measured in this beaker?

1. Find absolute uncertainty of the beaker

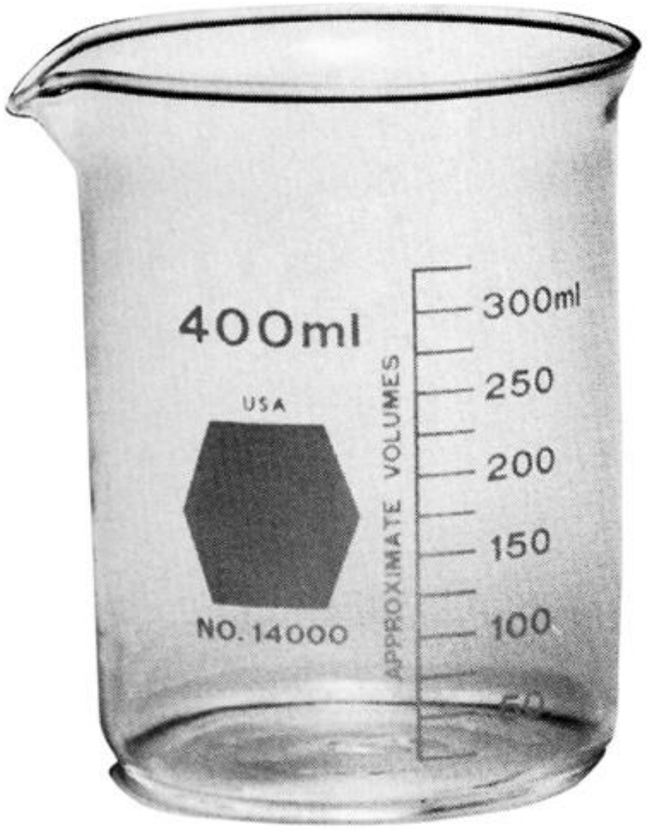
½ * 25 ml = 12.5 ml = absolute uncertainty

2. Calculate relative uncertainty

$$\frac{12.5\ ml}{100\ ml} = 0.125$$

- **Example 3:** What is the relative uncertainty of 300 ml?

$$\frac{12.5\ ml}{300\ ml} = 0.042$$



Absolute Uncertainty and Error Propagation

- **Addition and Subtraction** of multiple measurements with the same units (ex. perimeter):

1. Calculate measurement by adding or subtracting

2. Add absolute uncertainties

- How can you tell if it's simple addition or subtraction? The <u>units do not change</u>. If you start with cm you could convert your final answer and also end with cm.

Example: You have a jug of water with an unknown amount of water. You have two beakers a 500 ml beaker with increments every 50 ml and a 100 ml beaker with increments every 20 ml. You fill the 500 ml beaker 3 times and you fill the 100 ml beaker 4 times.

1. How much water is in the jug?

2. What is the absolute uncertainty of the water in the jug (add absolute uncertainties)?

- How can you tell it's addition/subtraction?

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1. How much water is in the jug? 1900 ml
2. What is the absolute uncertainty of the water in the jug (add absolute uncertainties)?

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1. How much water is in the jug? 500ml + 500ml + 500ml + 100ml + 100ml +100ml +100ml = 1900 ml

or 3*500ml + 4*100ml = 1900ml

2. What is the absolute uncertainty of the water in the jug (add absolute uncertainties)?

3 * 25 ml + 4 * 10 ml = 115 ml

- How can you tell it's addition/subtraction?

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3 * 25 ml + 4 * 10 ml = 115 ml

- How can you tell it's addition/subtraction? Everything is in ml! Units do not change and even if you decided to convert to liters you could convert back to ml. If you wanted to express the volume of water in the jug with uncertainty it would be 1900 ± 115 ml.

Relative Uncertainty and Error Propagation

- **Multiplication or Division** of multiple measurements (area, volume, density, etc..):

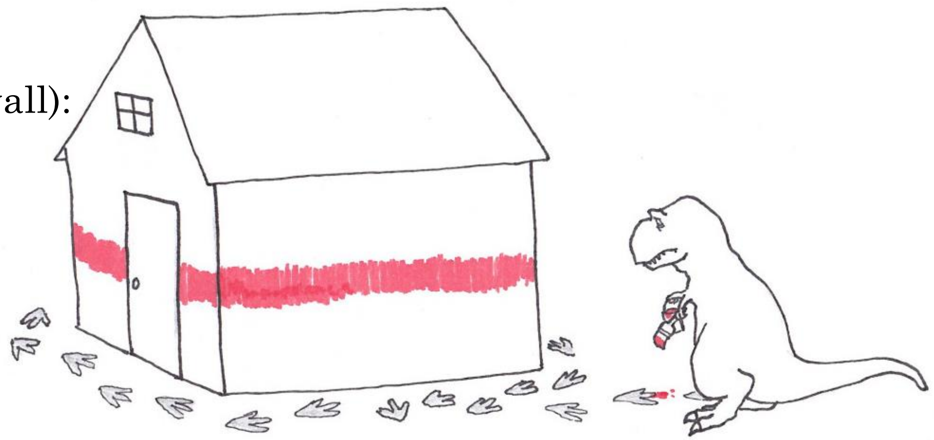
1. Calculate the final answer without uncertainties
2. Calculate relative uncertainties for each measurement
3. Add relative uncertainties
4. Multiply the sum of the relative uncertainties by your final answer to get the absolute uncertainty of your area, volume, density, etc.

- How can you tell if it's multiplication or division? The <u>units change</u> and you cannot simply convert the units using powers of 10 to get the units with which you started.

Relative Uncertainty and Error Propagation

- **Example of Multiplication/Division** of multiple measurements (area, volume, density, etc..): T-Rex is trying to paint his house but can't reach certain places. If T-Rex, lives in a square house with walls that are 21m ± 0.5m long and his completed swath of paint is 6m ± 0.5m wide, what area has he painted on one wall?

- What did T-rex use to measure his house?



1. Calculate final answer without uncertainties (area of paint on one wall):
2. Calculate relative uncertainties for each swath measurement
   -
   -
3. Add relative uncertainties
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4. Multiply the sum of the relative uncertainties by your final answer to get the absolute uncertainty of your area, volume, density, etc.
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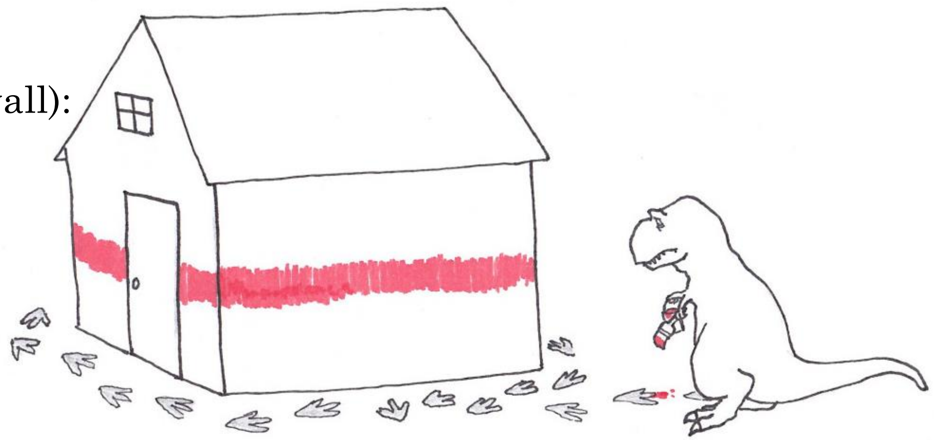
- What did T-rex use to measure his house? A meter stick



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1. Calculate final answer without uncertainties (area of paint on one wall): 6m * 21 m = 126 m^2

2. Calculate relative uncertainties for each swath measurement
   - Height: 0.5m / 6m = 0.08
   - Width: 0.5m / 21m = 0.02

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   -

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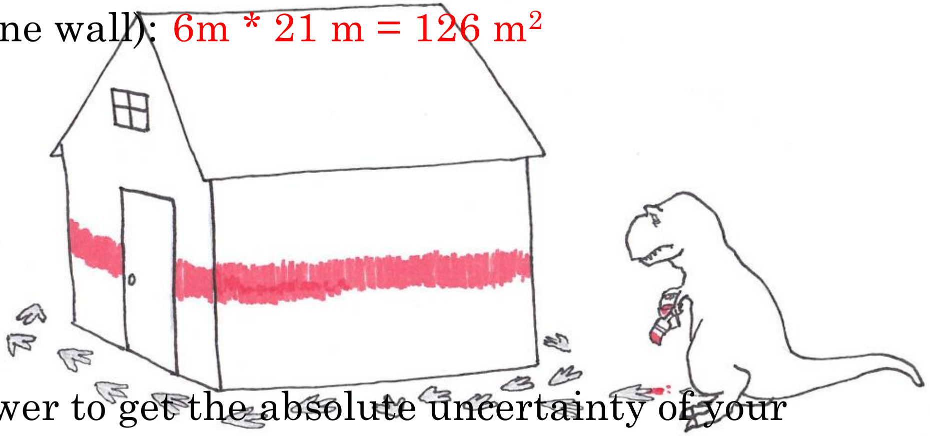
   -



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   - 0.08 + 0.02 = 0.10
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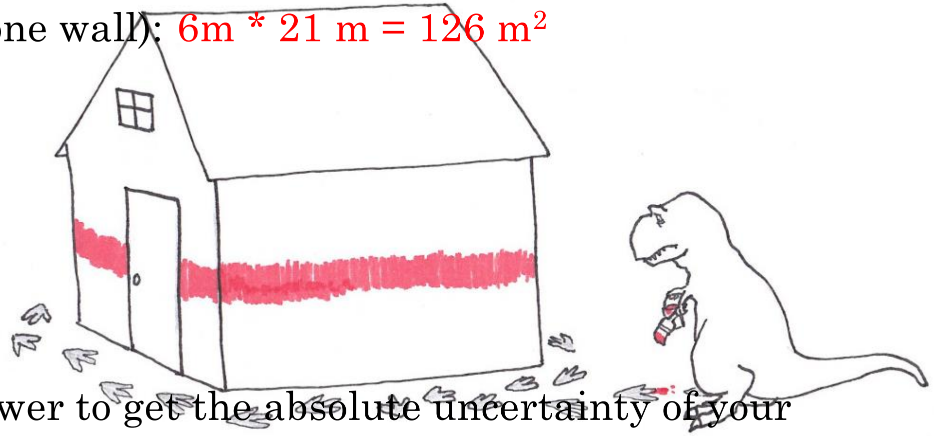

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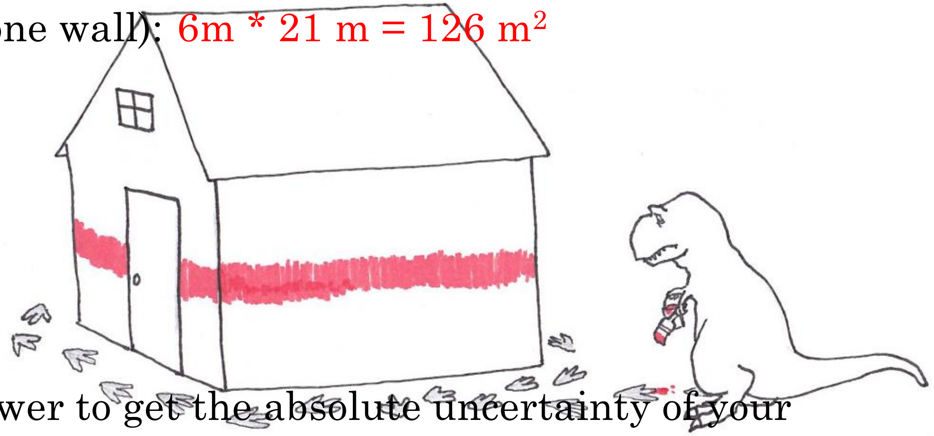

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2. Calculate relative uncertainties for each swath measurement
   - Height: 0.5m / 6m = 0.08
   - Width: 0.5m / 21m = 0.02

3. Add relative uncertainties
   - 0.08 + 0.02 = 0.10

4. Multiply the sum of the relative uncertainties by your final answer to get the absolute uncertainty of your area, volume, density, etc.
   - 6 m * 21 m = 126 m^2 * 0.10 = 12.6m^2 = 13 m^2
   - T-Rex painted 126 ± 13 m^2

- How can you tell if it's multiplication or division? The units change and you cannot simply convert the units using powers of 10 to get the units with which you started.

Relative Uncertainty and Error Propagation

- **Example of Multiplication/Division** of multiple measurements (area, volume, density, etc..): T-Rex is trying to paint his house but can't reach certain places. If T-Rex, lives in a square house with walls that are 21m ± 0.5m long and his completed swath of paint is 6m ± 0.5m wide, what area has he painted on one wall?

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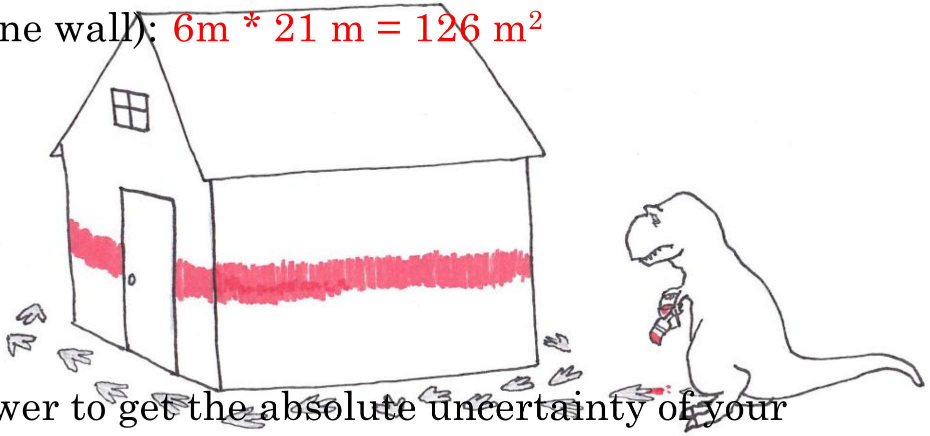

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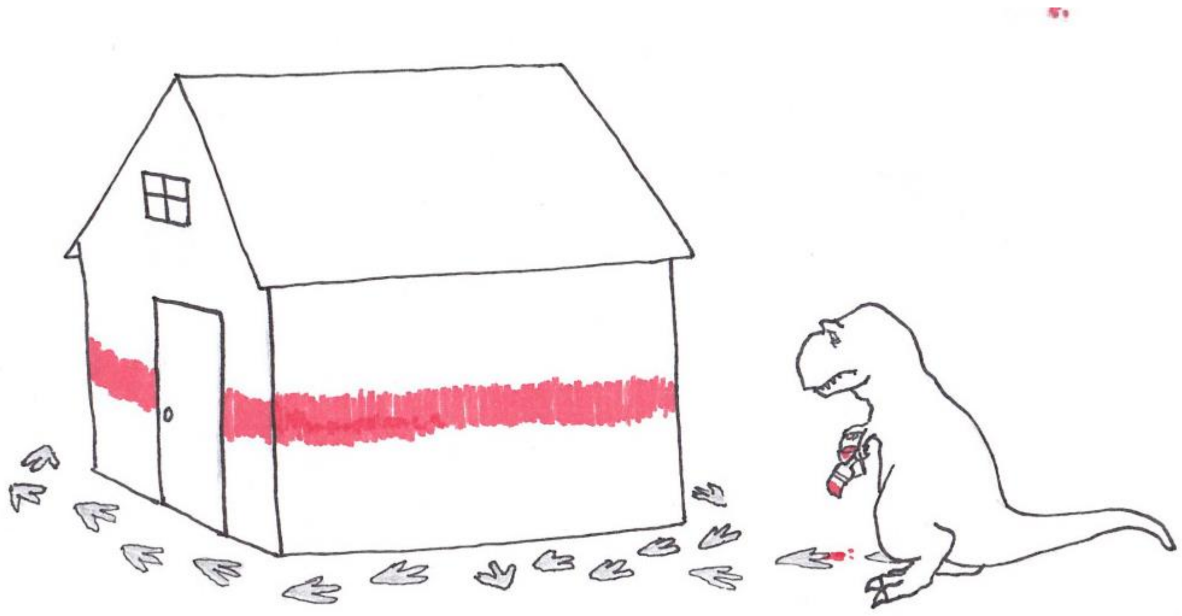
   - 6 m * 21 m = 126 m^2 * 0.10 = 12.6m^2 = 13 m^2
   - T-Rex painted 126 ± 13 m^2

- How can you tell if it's multiplication or division? The <u>units change</u> and you cannot simply convert the units using powers of 10 to get the units with which you started.
   - Started with m and ended with m^2 which you cannot convert back to m by multiplying by a multiple of 10

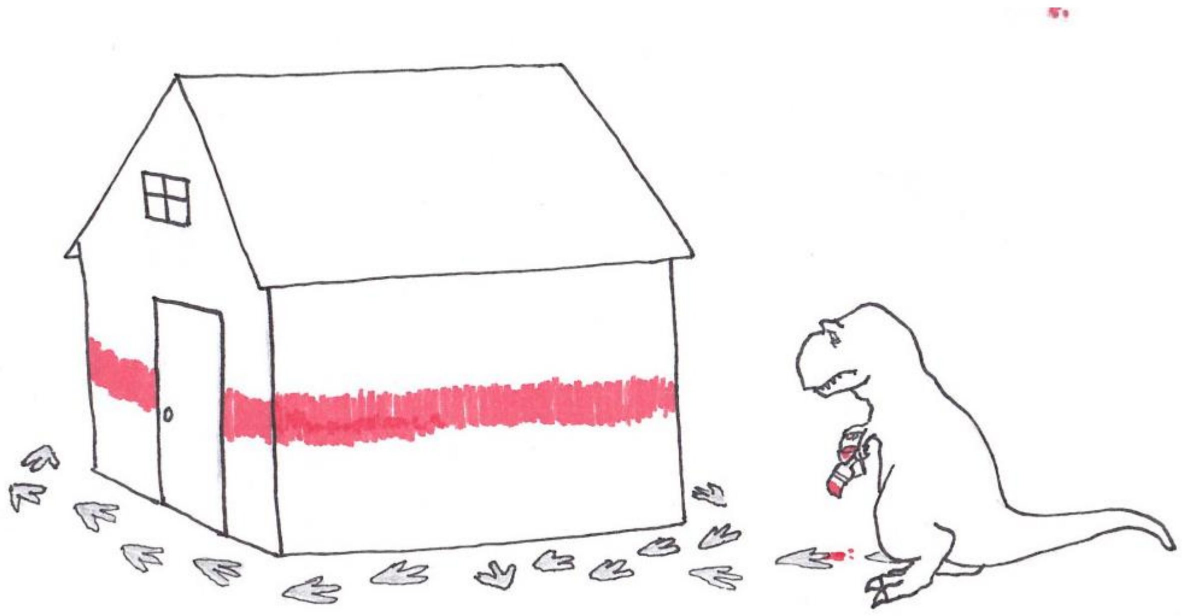
Uncertainty and Error Propagation

- What area has he painted on all four walls?



Absolute Uncertainty and Error Propagation

- What area has he painted on all four walls?
- *Simple addition*
- If one wall: 126 ± 13 m^2
- Then four walls: 4 * (126 ± 13 m^2)
- 504 ± 52 m^2



Types of Uncertainties:

1. Absolute
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3. **Percent**
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Percent Uncertainty

- Percent Uncertainty = Relative Uncertainty *100
- Recall: $\text{Relative Uncertainty} = \dfrac{\text{Absolute Uncertainty}}{\text{Measurement}}$
- Example: What is the percent uncertainty of a narwhal of length 6.4 m if the absolute uncertainty is 6.2 cm (0.062 m)?
- Standard form: 6.4 ± 0.062 m
- Calculate percent uncertainty:

$$\frac{0.062\text{ m}}{6.4\text{ m}} * 100 = 0.97\%$$

Types of Uncertainties:

1. Absolute
2. Relative or Fractional
3. Percent
4. **Min-Max**

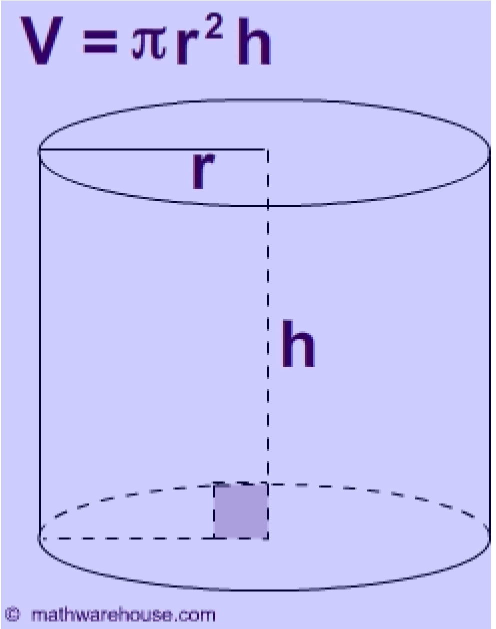
Min-Max Uncertainty (just FYI)
You will not be tested on this!

1. Calculate the minimum measurement
2. Calculate the maximum measurement
3. Divide the difference between the minimum measurement and maximum measurement by two to get the min-max uncertainty

Min-Max Uncertainty

- Example: You have a circular shark pool with a radius of 1 ± 0.05 m and a height of 35 ± 1 cm, how much water (with min-max uncertainty) will you need to fill the pool?

0. Convert everything to meters: Height: 0.35 ± 0.01 m



1. Calculate minimum volume:

   Radius: 1m - 0.05m = 0.95 m,

   Height: 0.35m - 0.01m = 0.34 m

   $V_{min} = \pi(0.95m)^2 * 0.34m = 0.96\ m^3$

2. Calculate maximum volume

   Radius: 1m + 0.05m = 1.05 m,

   Height: 0.35m + 0.01m = 0.36 m

   $V_{max} = \pi(1.05m)^2 * 0.36m = 1.25\ m^3$

3. Divide the difference between the minimum volume and the maximum volume:

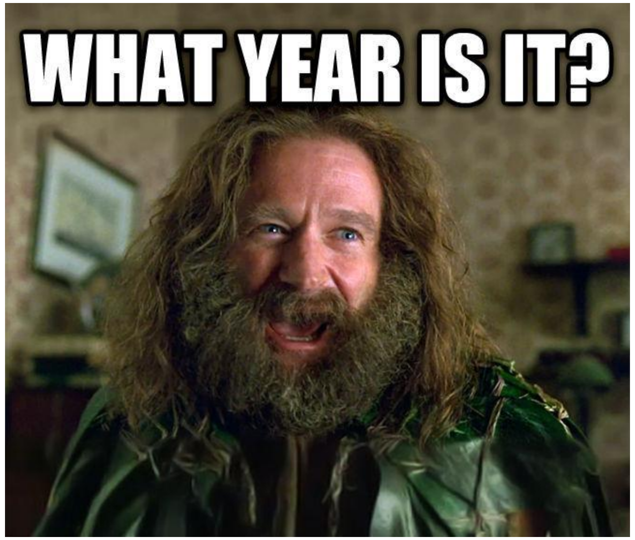
   $\frac{1.25\ m^3 - 0.96\ m^3}{2} = 0.15\ m^3$ so you would need 1.10 ± 0.15 m^3 to fill the pool

Summary

1. **Absolute Uncertainty= ½ * smallest increment on measuring device**

2. $$\text{Relative Uncertainty} = \frac{\text{Absolute Uncertainty}}{\text{Measurement}}$$

3. Percent Uncertainty = Relative Uncertainty *100



Propagating Uncertainty/Error: Rules of Thumb

Use Absolute Uncertainty if all of your measurements and their associated uncertainties have the same units (keep in mind if you can multiply by an order of magnitude to get the same unit this still counts as the same…Example: given two volumes, 1.25 ± 0.01 L and 850 ± 1 ml find the total volume and uncertainty CONVERT 850 ± 1 ml = 0.85 ± 0.01 L…total = 2.10 ± 0.02 L)

Use Relative Uncertainty if your measurements and their associated uncertainties DO NOT have the same units.

Use Percent Uncertainty if your measurements and their associated uncertainties DO NOT have the same units AND the uncertainty is very small relative to the measurement AND there are few significant figures. We prefer to keep the correct number of decimal places as dictated by sig figs.

Which Uncertainty Should I Use?

- The mass of a wooden block is 600.0 ± 0.1g The volume of the block is 1000 ± 5ml, what is the density?

- The length of a table is 4.00 ± 0.01m and the width is 3.00 ± 0.01m, what is the area?

- A ruler measuring mm is used to find the perimeter of a rectangular tile. The longer sides of the tile measure 625mm and the shorter sides measure 42.5cm, what is the perimeter of the tile?

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  - Percent Uncertainty would be the best choice but relative would also work

- The length of a table is 4.00 ± 0.01m and the width is 3.00 ± 0.01m, what is the area?
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- A ruler measuring mm is used to find the perimeter of a rectangular tile. The longer sides of the tile measure 625mm and the shorter sides measure 42.5cm, what is the perimeter of the tile?
  - Absolute Uncertainty is the only choice

Which Uncertainty Should I Use?

- The mass of a wooden block is 600.0 ± 0.1g The volume of the block is 1000 ± 5ml, what is the density?
  - Percent Uncertainty would be the best choice but relative would also work
  - Percent Answer: 0.6 g/ml ± 0.3% (Relative: 0.6 ± 0.003 g/ml—not preferable)
- The length of a table is 4.00 ± 0.01m and the width is 3.00 ± 0.01m, what is the area?
  - Relative Uncertainty would be the best choice but percent would also work
  - Relative Answer: 12.00 ± 0.07 m^2 (Percent: 12.00m^2 ± 7.00%—we'd prefer to keep our units)
- A ruler measuring mm is used to find the perimeter of a rectangular tile. The longer sides of the tile measure 625mm and the shorter sides measure 42.5cm, what is the perimeter of the tile?
  - Absolute Uncertainty is the only choice
  - Absolute Answer: 2100 ± 4mm or 210.0 ± 0.4cm

Average, Standard Deviation, & Range

Average ($\bar{x}$)

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i = \frac{1}{n}(x_1 + x_2 + x_3 + \cdots + x_n)$$

- Sigma (Σ) means add up (sum)
- *n* is the number of items/numbers in series (above the sigma it dictates when to stop performing the action in front of sigma—simple summation)
- *i*=1 means start summing with the first number in the series because i tells us with which number in the series we should start summing…if i=2 we would sum like this: $x_2 + x_3 + x_4$ +…
- 1/n is before Σ so we know to add starting with the first number (i=1) and stop summing when we reach the last number, n, (top of sigma tell us at which number in the series we should stop summing) and multiply that entire sum by 1/n

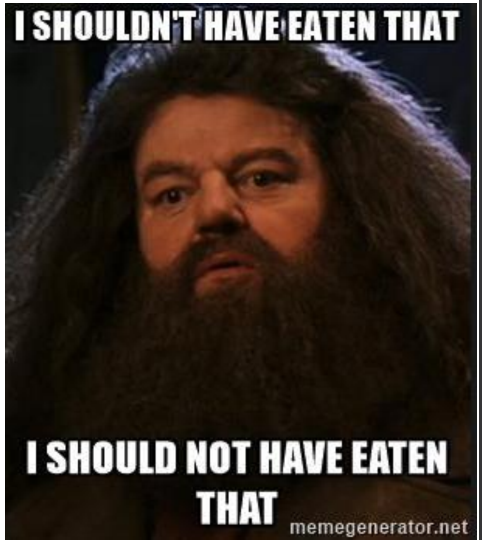
Average ($\bar{x}$)

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i = \frac{1}{n}(x_1 + x_2 + x_3 + \cdots + x_n)$$

- Sigma (Σ) means add up (sum)
- n is the total number of items
- i=1 means start adding with the first number in the series

Example: Hagrid measures the lengths of 6 unicorn horns. The lengths in cm are as follows: 26.1, 22.3, 24.5, 20.9, 25.2, and 27.0. What is the average length of horn for these six unicorns?

$$\frac{26.1 + 22.3 + 24.5 + 20.9 + 25.2 + 27.0}{6} = \frac{146.0}{6} = 24.3 \text{ cm}$$



Standard Deviation

Represents variation or uncertainty in a series of numbers

Sample Standard Deviation:

$$s = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}}$$

- Sigma (Σ) means add up (sum)
- n is the total number of items/numbers in a series (above the sigma it dictates when to stop performing the action in front of sigma)
- i=1 means start adding with the first number in the series
- x_i is any single number in a series of numbers
- $\bar{x}$ is the average of that series of numbers

Population Standard Deviation:

$$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \mu)^2}$$

- μ is the population mean
- Use this only when dealing with the entire population of measurements

Sample Standard Deviation

Example: Hagrid measures the lengths of 6 unicorn horns the lengths in cm are as follows: 26.1, 22.3, 24.5, 20.9, 25.2, and 27.0. What is the standard deviation/how much variation is there in unicorn horn length?

Sample Standard Deviation:

$$s = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}}$$

- Sigma (Σ) means add up (sum)
- n is the total number of items in a series
- i=1 means start adding with the first number in the series
- x_i is a single number in a series of numbers
- $\bar{x}$ is the average of that series of numbers (sample mean)

What is the standard deviation?

Sample Standard Deviation

Example: Hagrid measures the lengths of 6 unicorn horns the lengths in cm are as follows: 26.1, 22.3, 24.5, 20.9, 25.2, and 27.0. What is the standard deviation/how much variation is there in unicorn horn length?

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- x_i is a single number in a series of numbers
- $\bar{x}$ is the average of that series of numbers (sample mean)

$$s = \sqrt{\frac{(26.1-24.3)^2 + (22.3-24.3)^2 + (24.5-24.3)^2 + (20.9-24.3)^2 + (25.2-24.3)^2 + (27.0-24.3)^2}{6-1}} =$$

$$\sqrt{\frac{(1.8)^2 + (-2.0)^2 + (0.2)^2 + (-3.4)^2 + (0.9)^2 + (2.7)^2}{5}} = \sqrt{\frac{3.24 + 4 + 0.04 + 11.56 + 0.81 + 7.29}{5}} = \sqrt{\frac{26.94}{5}} = \sqrt{5.4} = 2.3 \text{ cm}$$

How should you express the standard deviation/uncertainty?

Sample Standard Deviation

Example: Hagrid measures the lengths of 6 unicorn horns the lengths in cm are as follows: 26.1, 22.3, 24.5, 20.9, 25.2, and 27.0. What is the standard deviation/how much variation is there in unicorn horn length?

Sample Standard Deviation:

$$s = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}}$$

- Sigma (Σ) means add up (sum)
- n is the total number of items in a series
- i=1 means start adding with the first number in the series
- x_i is a single number in a series of numbers
- $\bar{x}$ is the average of that series of numbers (sample mean)

$$s = \sqrt{\frac{(26.1-24.3)^2 + (22.3-24.3)^2 + (24.5-24.3)^2 + (20.9-24.3)^2 + (25.2-24.3)^2 + (27.0-24.3)^2}{6-1}} =$$

$$\sqrt{\frac{(1.8)^2 + (-2.0)^2 + (0.2)^2 + (-3.4)^2 + (0.9)^2 + (2.7)^2}{5}} = \sqrt{\frac{3.24 + 4 + 0.04 + 11.56 + 0.81 + 7.29}{5}} = \sqrt{\frac{26.94}{5}} = \sqrt{5.4} = 2.3 \text{ cm}$$

How should you express the standard deviation/uncertainty? 24.3 ± 2.3 cm

Population Standard Deviation

Example: Hagrid measures the lengths of 6 unicorn horns (the only 6 unicorns in existence/to ever be in existence) the lengths in cm are as follows: 26.1, 22.3, 24.5, 20.9, 25.2, and 27.0. What is the standard deviation?

Population Standard Deviation:

$$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \mu)^2}$$

- Sigma (Σ) means add up (sum)
- n is the total number of items in a series
- i=1 means start adding with the first number in the series
- x_i is a single number in a series of numbers (sample measurement)
- μ is the average of all numbers in a particular series (population mean)

NEVER USE THE EXCEL FUNCTION STDEV.P for physics labs

Population Standard Deviation

Example: Hagrid measures the lengths of 6 unicorn horns (the only 6 unicorns in existence/to ever be in existence) the lengths in cm are as follows: 26.1, 22.3, 24.5, 20.9, 25.2, and 27.0. What is the standard deviation? (Note that since these are the only unicorns EVER 24.3 is now the population mean, μ.)

$$\sigma = \sqrt{\frac{(26.1-24.3)^2 + (22.3-24.3)^2 + (24.5-24.3)^2 + (20.9-24.3)^2 + (25.2-24.3)^2 + (27.0-24.3)^2}{6}} =$$

$$\sqrt{\frac{(1.8)^2 + (-2.0)^2 + (0.2)^2 + (-3.4)^2 + (0.9)^2 + (2.7)^2}{6}} = \sqrt{\frac{3.24 + 4 + 0.04 + 11.56 + 0.81 + 7.29}{6}} =$$

$$\sqrt{\frac{26.94}{6}} = 2.1 \text{ cm}$$

How should Hagrid express the standard deviation/uncertainty? 24.3 ± 2.1 cm

Range

$\bar{x} + s = \text{maximum}$

$\bar{x} - s = \text{minimum}$

Example: Dumbledore has a 30 cm standard Owl Post box to ship a unicorn horn so he asks Hagrid if the box will be large enough. What is the range of unicorn horn lengths? Is the box large enough?

- 24.3 ± 2.3 cm so between
  - 24.3 – 2.3 = 22.0 and
  - 24.3 + 2.3 = 26.6
- Yes, the box is large enough because 30>26.6

Percent Error vs Percent Difference

Percent Error (Uncertainty)

Percent error compares an experimental value to a known or theoretical value.

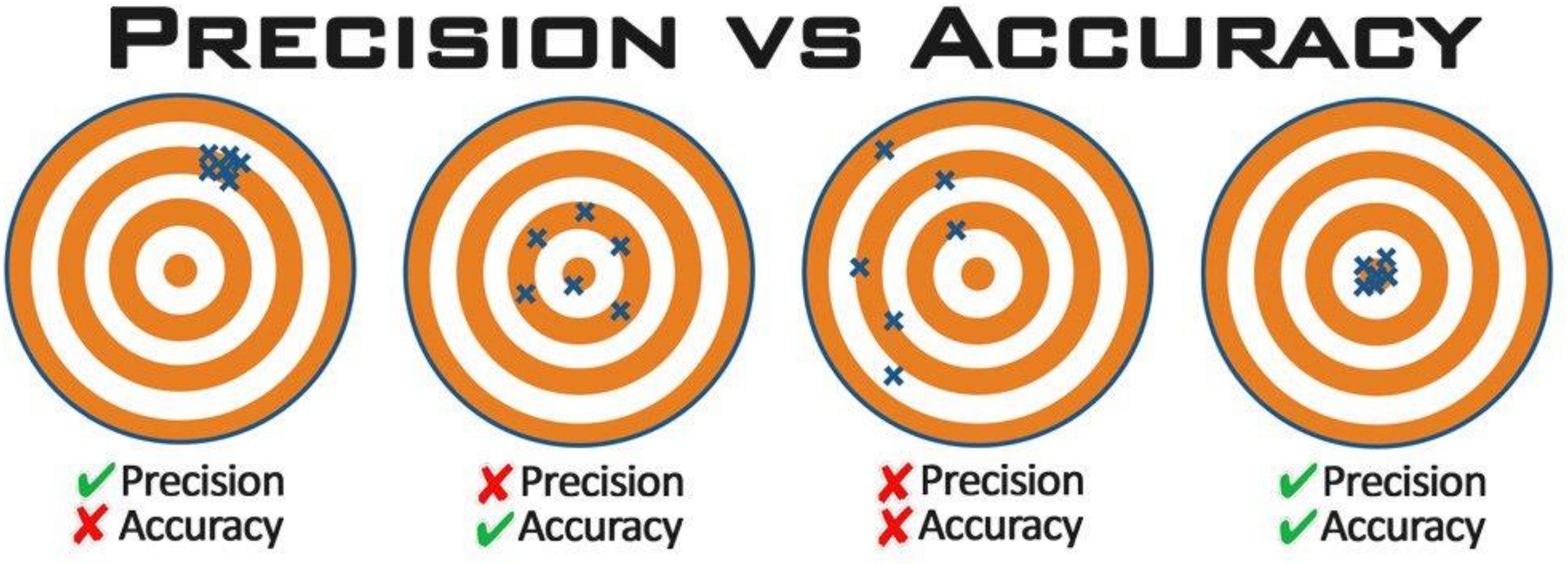
$$\%\text{Error} = \frac{|Theoretical\ Value - Experimental\ Value|}{Theoretical\ Value} * 100$$

Percent Difference

Percent difference compares two experimental values.

$$\%\text{Difference} = \frac{|Experimental\ Value\ 1 - Experimental\ Value\ 2|}{\frac{1}{2}(Experimental\ Value\ 1 + Experimental\ Value\ 2)} * 100$$

Accuracy vs Precision



- Small standard deviation
- Small range
- Large %Error

- Large standard deviation
- Large range
- Small %Error

- Large standard deviation
- Large range
- Large %Error

- Small standard deviation
- Small range
- Small %Error

http://forums.gunsandammo.com/showthread.php?31538-Precision-Vs-Accuracy