Atwood Machine

Equipment

- Computer, PASCO Interface
- Table Clamp
- Double pulley apparatus (one smart pulley)
- Smart Pulley Data Cable
- String
- Two Mass Hangers
- One Mass Set (1×500, 2×200, 1×100, 1×50, 2×20, 1×10, 1×5, 2×2, 1×1)
- Styrofoam Pad

Objectives

- To verify Newton’s Second Law as applied to an Atwood Machine.

Introduction

According to Newton’s 2nd Law the acceleration of an object depends on the net applied force and the object’s mass.

\[ \sum \vec{F} = ma \]  

In an Atwood Machine, there are two objects, each one is a mass hanging from one end of a string. The string is hung over a pulley so that both masses are hanging. The weight of one mass pulls the system in one direction, while the weight of the other mass pulls in the other direction.

\[ \begin{align*}
F_{g1} &= m_1 g \\
F_{g2} &= m_2 g
\end{align*} \]

Taking the convention that up is positive and down is negative, the net force equations for \( m_1 \) and \( m_2 \) are:

Figure 1. Individual free body diagrams of each of the hanging masses.
\[ \sum F_{1y} = T - m_1g = m_1a_1 \quad \quad (2) \]
\[ \sum F_{2y} = T - m_2g = m_2a_2 \quad \quad (3) \]

The key to solving this is to realize that in the system of \( m_1 \) and \( m_2 \), one mass accelerates upward while the other accelerates downward. So, for mass \( m_2 \), we flip the coordinates and then the accelerations of \( m_1 \) and \( m_2 \) are the same and we can just call them both \( a \).

\[ \sum F_{1y} = T - m_1g = m_1a \quad \quad (4) \]
\[ \sum F_{2y} = -T + m_2g = m_2a \quad \quad (5) \]

We can add equations (4) and (5) to obtain:

\[ m_2g - m_1g = m_1a + m_2a \]
\[ (m_2 - m_1)g = m_{tot}a \]

Solving for \( a \), the acceleration of the system of both masses, the theoretical acceleration is \( g \) times the difference in mass divided by the total mass:

\[ a = g \left( \frac{m_2 - m_1}{m_{tot}} \right) \]

If \( m_2 > m_1 \), then \( F_{g2} \) is stronger, and the net force on the system accelerates \( m_2 \) downward and \( m_1 \) upward.

**Treating the Atwood Machine as a System.**

Notice that Eq. 7 looks a lot like Newton’s 2nd Law (Eq. 1). The “object” is the pair of masses attached to either end of a string. On the right side of Eq. 7, the mass of the system is the total mass of the two hangers. The acceleration of the system is directly proportional to the exerted force and inversely proportional to the mass of the system.

On the left of Eq. 7, we have the net force on the system \( \Sigma F = (M - m)g \). The net force acting on the system of both masses is just the difference in weight between two hanging masses. This net force accelerates both of the hanging masses. The heavier mass is accelerated downward, and the lighter mass is accelerated upward.
**Experimental Plan**

Newton’s 2nd Law can be verified by setting up the Atwood Machine under various conditions. By carefully selecting the masses, we can graph:

(1) $\Sigma F$ vs. $a$ while keeping the system mass $\Sigma m$ constant.
(2) $a$ vs. $1/\Sigma m$ while keeping the net force, $\Sigma F = (M-m)g$, constant.

In graph (1), the result should be linear with the slope of the graph equal to the total mass. In graph (2), the result should be linear with the slope of the graph equal to the exerted net force $(M-m)g$. If this is true, then Newton’s 2nd Law is verified.

**Experimental Setup**

We will use the Photogate/pulley system to measure the motion of both masses as one moves upward and the other moves downward. Capstone software will record the changing speed of the masses as they move. The recorded data will be displayed as a velocity vs. time graph, where the slope of the graph is the acceleration of the system. Capstone file “Atwood’s Machine” is posted on Blackboard in a corresponding folder and should be open before starting the experiment.

**Hardware Setup**

1. Mount the universal clamp to the edge of the table.
2. Insert the connecting rod of the pulley into the clamp.
3. Plug the photogate into digital channel 1 of the interface. You can tell it’s working by spinning the pulley. A little red light near the pulley should blink.
4. Use a piece of string just long enough so that when one mass is on the floor/table, the other is a few centimeters below its pulley.
5. Hang mass hangers from each end of the string, and hang the string over the pulleys, as shown in Figure 2.

![Figure 2](Image source: PASCO)

6. Make sure the string isn’t rubbing against anything, such as the data cable.
Part 1: Constant Total Mass

1. Place 200g and 100g on a hanger that is designated as a larger mass, M.
2. Place 200g, 50 g, and another 50g composed from small weights (20+20+10) on other hanger, designated as a smaller mass m. This way both hangers have 350g and \( \sum m = 700g \).
3. While keeping \( \sum m \) the same, move 10g weight from m to M.
4. Move the masses so that \( M \) is hanging near the top and \( m \) is on the Styrofoam pad.
5. When you are ready to start, press Start in Capstone and release system.
6. When \( M \) reaches the bottom, press Stop.
7. Fill in the columns of the Table 1.
   - Run #: Record the run number from Capstone, in case you should come back to it.
   - \( m \), and \( M \): The individual masses including the hanger.
   - \( a \): the experimental acceleration, recorded by Capstone.
   - \( \sum F = (M - m)g \): net force acting on the system.
   - \( \sum m \): the total mass.

<table>
<thead>
<tr>
<th>Run #</th>
<th>( m ) (kg)</th>
<th>( M ) (kg)</th>
<th>( a ) (m/s²)</th>
<th>( \sum F ) (N)</th>
<th>( \sum m ) (kg)</th>
</tr>
</thead>
<tbody>
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</tbody>
</table>

Table 1. Example table for recording run parameters, results, and calculations for Part 1.

8. Move masses around in increments of 10g (place 10g back to \( m \) while moving 20g to \( M \), then 30g, 40g, and, finally 50g) while collecting data for each run and recording it in Table 1.

9. Plot \( \sum F \) vs. \( a \) and fit it into linear trendline. The slope of the linear trendline should be the total mass of the system. Record your results in Table 2.

<table>
<thead>
<tr>
<th>Total Mass (kg)</th>
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</table>

<table>
<thead>
<tr>
<th>Slope of ( \sum F ) vs. ( a ) (kg)</th>
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<tbody>
<tr>
<td></td>
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</table>

<table>
<thead>
<tr>
<th>% Difference</th>
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<tbody>
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</table>

Table 2. Experimental and expected total mass.
Part 2: Constant Net Force

1. Leave 200g and 100g on the hanger that is designated as a larger mass, M.
2. Leave only 200g on the hanger, designated as a smaller mass m. This way the weight difference between the hangers is 100g and \( \sum F = 0.1 \text{kg} \times 9.8 \frac{\text{m} \cdot \text{s}^2}{\text{kg}} \).
3. Move the masses so that \( M \) is hanging near the top and \( m \) is on the Styrofoam pad.
4. When you are ready to start, press Start in Capstone and release system.
5. When \( M \) reaches the bottom, press Stop.
6. Fill in the columns of the Table 3.
   - Run #: Record the run number from Capstone, in case you should come back to it.
   - \( m \), and \( M \): The individual masses including the hanger.
   - \( a \): the experimental acceleration, recorded by Capstone.
   - \( \sum F = (M - m)g \): net force acting on the system.
   - \( \sum m \): the total mass.

<table>
<thead>
<tr>
<th>Run #</th>
<th>( m ) (kg)</th>
<th>( M ) (kg)</th>
<th>( 1/\sum m ) (kg)</th>
<th>( a ) (m/s(^2))</th>
<th>( \sum F ) (N)</th>
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</table>

**Table 3.** Example table for recording run parameters, results, and calculations for Part 2.

7. While keeping \( \sum F \) the same, increase weight of each hanger by increments of 10g (one 10g set comes from smaller weights: 5+2+2+1) while collecting data for each run and recording it in Table 3.
8. Plot \( a \) vs. \( \frac{1}{\sum m} \) and fit it into linear trendline. The slope of the linear trendline should be the net force acting on the system. Record your results in Table 4.

<table>
<thead>
<tr>
<th>Net Force (N)</th>
<th></th>
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<tbody>
<tr>
<td>Slope of ( a ) vs. ( \frac{1}{\sum m} ) (N)</td>
<td></td>
</tr>
<tr>
<td>% Difference</td>
<td></td>
</tr>
</tbody>
</table>

**Table 4.** Experimental and expected net force.
Requirements for the Report (also consult the rubric):

- Save your Excel files through your Blackboard Group File Exchange
- The lab report must include 4 Tables and 2 graphs, all items with captions explaining the content of the table/graph.
- The abstract must contain an explanation
  - how the Data was collected and calculated for Table 1
  - how the Data from Table 1 was analyzed including interpretation of the trendline and comparison of values in Table 2
  - how the Data was collected and calculated for Table 3
  - how the Data from Table 3 was analyzed including interpretation of the trendline and comparison of values in Table 4
  - A general statement based off Tables 2 and 4 about how the experimental results support Second Law of Motion