Interference of Light Waves

Equipment and Safety:
• Do not look directly into the laser. Do not point the laser at other people.
• Be careful about reflections. It’s easy to accidentally send a laser reflection into someone’s eye.
• Laser
• Diffraction Grating
• Grating holder and stand
• Meter stick and ruler
• Metal Washer for mounting a hair.

Objectives
• To measure the wavelength of laser light using a diffraction grating.
• To measure the thickness of a piece of human hair using interference of laser light.

Introduction
Light can be treated in terms of "rays" that propagate in straight lines for many circumstances. However, light has a dual character and as a result, when the ray model fails, the wave nature of light is of central importance. The wave properties of light are responsible for such phenomena as the operation of a CD, the appearance of images on a TV screen, and the brilliant iridescent colors of a butterfly's wing. If the dimensions of objects encountering light are large compared to the wavelength of light, light can be treated as a bundle of rays, each ray being a line along which light energy flows. If the dimensions are comparable with the wavelength of light, then the wave theory of light explains the observed optical effects.

The simplest example of diffraction is called the two-slit experiment. A beam of light hits a barrier that blocks most of the light. There are, however, two small slits in the barrier that allow light to pass through. Light propagating from the two slits to a distant screen along parallel paths makes an angle $\theta$ with respect to the normal of the slits. The difference in path length is $d \sin \theta$, where $d$ is the slit separation.

Figure 1. Geometry of the two-slit experiment, including the path length difference $\delta$.
(Source: Serway/Jewett, Physics for Scientists & Engineers, 9th Ed.)
If the path length difference is an integer number of wavelengths, there is constructive interference. So, the conditions for Bright Fringes (Constructive Interference) are:

\[ d \sin \theta = m\lambda \quad \text{Constructive if } m = 0, \pm 1, \pm 2, \ldots \]

If the path length difference is in halfway in between integer multiples of the wavelength, there is destructive interference. So, the conditions for Dark Fringes (Destructive interference) are:

\[ d \sin \theta = m\lambda \quad \text{Destructive if } m = \pm 0.5, \pm 1.5, \pm 2.5, \ldots \]

Usually, the angle \( \theta \) is measured by measuring a large triangle. The adjacent side is \( L \) and the opposite side is \( y \) in Figure 1. So the angle is found from:

\[ \tan \theta = \frac{y}{L} \]

A diffraction grating is just a series of slits in a barrier. In practice, this is usually done by starting with a transparent film and drawing lines or scratching grooves. As above, we call the spacing \( d \). The difference in path length for rays from neighboring slits is the same: \( d \sin \theta \)

![Figure 2. Geometry of the path length difference of a diffraction grating. (Source: Serway/Jewett, Physics for Scientists & Engineers, 9th Ed.)](image)

The conditions for Constructive Interference in a diffraction grating are the same as in the two-slit experiment. With the diffraction grating, any time \( m \) is not an integer, there is destructive interference.

A diffraction grating is often characterized in terms of the line density. This is the number of lines per unit length, \( N \). For example, a particular grating might have 2250 lines/cm. The corresponding slit separation, \( d \) is simply the inverse of the line density.

\[ d = \frac{1}{N} \]

In the example, the line spacing is \( d = 1/(2250 \text{ lines/cm}) = 0.000444 \text{ cm} = 4.44 \mu\text{m} \).

![Figure 3. Geometry of single-slit diffraction. (Source: Serway/Jewett, Physics for Scientists & Engineers, 9th Ed.)](image)

**Single-Slit Diffraction:** When light of a single wavelength \( \lambda \) passes through a slit of width \( a \), a diffraction pattern of bright and dark fringes is formed.
The mechanism behind single-slit diffraction is explained in the textbook. The conditions for Dark Fringes in a Single-Slit Experiment are very similar to the two-slit equations:

\[ m\lambda = a \sin \theta \quad \text{Destructive if } m = \pm 1, \pm 2, \pm 3, \ldots \]

Notice that the \( m = 0 \) case is the central bright area. That’s just the light that goes straight through.

Why do we care about single-slit diffraction? Mainly because it can be used to find the size of small objects. This is because of Babinet’s principle.

**Babinet's principle**: A theorem concerning diffraction that states that the diffraction pattern from an opaque body is identical to that from a hole of the same size and shape except for the overall forward beam intensity. The principle is most often used in optics but it is also true for other forms of electromagnetic radiation and is a general theorem of diffraction and holds true for all waves. Babinet's principle finds most use in its ability to detect equivalence in size and shape. Diffraction patterns from apertures or bodies of known size and shape are compared with the pattern from the object to be measured. For instance, the size of red blood cells can be found by comparing their diffraction pattern with an array of small holes.

**Part #1. Diffraction Grating**

1. Shine a laser toward a screen.
2. Place a diffraction grating between the laser and the screen. Position it so you can see two bright spots on either side of the main laser spot.
3. Measure the distance \( L \) between the screen and the diffraction grating.
4. Measure position of each spot from the central bright spot, \( y_m \).
5. Record the values on the left side of the central spot as negative.

<table>
<thead>
<tr>
<th>Diffraction order ( m )</th>
<th>Position ( y_m ) from central spot (m)</th>
<th>( \sin \theta = \frac{y_m}{\sqrt{(y_m)^2 + L^2}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
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<tr>
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<td>0</td>
<td>0</td>
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<tr>
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<tr>
<td>−2</td>
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</tbody>
</table>

*Table 1.* Positions of several diffraction spots in the diffraction grating experiment

6. Plot \( \sin \theta \) vs. \( m \).
7. Use a linear fit to find the slope of the graph.
8. Calculate the slit separation \( d \) from the grating density given in Table 2.
9. Use the slope and the slit separation to find the wavelength \( \lambda \). **Hint:** \( \sin \theta = \frac{m\lambda}{d} \)
<table>
<thead>
<tr>
<th>Quantity (Units)</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance ( L ) (m)</td>
<td></td>
</tr>
<tr>
<td>Grating Density (lines/mm)</td>
<td>530</td>
</tr>
<tr>
<td>Grating Spacing ( d ) (m)</td>
<td></td>
</tr>
<tr>
<td>Slope of ( \sin \theta ) vs. ( m )</td>
<td></td>
</tr>
<tr>
<td>Wavelength ( \lambda ) (nm)</td>
<td></td>
</tr>
</tbody>
</table>

**Table 2.** Experimental results for the diffraction grating setup.

10. Look up the wavelengths that correspond to the color of the laser dots to assure that your calculated wavelength is reasonable.

**Part #2. Measuring the width of a hair.**

1. Shine a laser toward a screen.
2. Mount a human hair to one of the washers and place it in front of the laser beam. You should see the laser spot with small lines of light on either side, in a direction perpendicular to the hair.
3. Look closely at the lines of light to notice the dark fringes.
4. Measure the distance \( L \) between the screen and the washer.
5. Tape a sheet of paper to the screen and mark positions of the central spot and each dark fringe.
6. Measure position of each spot from the central bright spot, \( y_m \).
7. Record the values on the left side of the central spot as negative.

<table>
<thead>
<tr>
<th>Diffraction order ( m )</th>
<th>Position ( y_m ) from central spot (m)</th>
<th>( \sin \theta = \frac{y_m}{\sqrt{(y_m)^2 + L^2}} )</th>
</tr>
</thead>
<tbody>
<tr>
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<tr>
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<tr>
<td>-2</td>
<td></td>
<td></td>
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<tr>
<td>-3</td>
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</tbody>
</table>

**Table 1.** Positions of several diffraction spots in the hair experiment

8. Plot \( \sin \theta \) vs. \( m \).
9. Use a linear fit to find the slope of the graph
10. Use the slope and the wavelength \( \lambda \) to find the width of the hair, **Hint:** \( \sin \theta = \frac{m\lambda}{a} \)
11. The result should be within a range of normal hair thickness or between 25 to 135 \( \mu \)m.