## Comparing with Percentages

Using relative differences (i.e. percentages) rather than absolute differences is important because the significance of an absolute difference depends on the sizes of the quantities involved.  The percentage in question is always a **percentage of something**.

- Experimental Percent Error – In this case you're evaluating a new measured value as compared to a previously known theoretical or accepted value.  The "Error" is the absolute value of the difference between the two values.

$$\% \text{ Error} = \left(\frac{|\text{Error}|}{\text{Accepted}}\right)(100\%) = \left(\frac{|\text{Measured} - \text{Accepted}|}{\text{Accepted}}\right)(100\%)$$

- Percent Difference – In this case you have two values that are on equal footing.  Then there is no good way of choosing which belongs in the denominator.  As a compromise, place the average in the denominator.

$$\% \text{ Difference} = \left(\frac{|\text{Difference}|}{\text{Average}}\right)(100\%) = \left(\frac{|\text{Val2} - \text{Val1}|}{(\text{Val1} + \text{Val2})/2}\right)(100\%)$$

- Percent Change – In this case, some value is changing, and there is a new value and an old value.  Here, it may be interesting to note whether the value increased or decreased, so an absolute value isn't usually taken.  Assuming positive quantities, a positive change is an increase and a negative change is a decrease.

$$\% \text{ Change} = \left(\frac{\text{New} - \text{Old}}{\text{Old}}\right)(100\%)$$

Reference:

http://webassign.net/question_assets/ncsucalcphysmechl3/percent_error/manual.html

# Uncertainty

A measurement uncertainty is a possible error in a measurement. It's expected that the actual error in the measurement is less than the uncertainty. Uncertainty can be expressed as an absolute quantity (in the same units as the measurement) or as a relative quantity (i.e. a percentage). Uncertainty is always positive, even though we don't know which direction the error is in.

There are a few examples below.

## Propagation of Uncertainty

How do uncertainties combine to produce results with uncertainty? Generally, when adding or subtracting, add the uncertainties. When multiplying or dividing, add the relative uncertainties.

## Quick Examples of Uncertainty

Quantities can be expressed with absolute or relative uncertainty.
- $x = 50 \pm 1$ m $= 50$ m $\pm 2\%$
- $y = 20 \pm 1$ m $= 20$ s $\pm 5\%$
- $t = 25 \pm .5$ s $= 25$ s $\pm 2\%$

When adding or subtracting, add the uncertainties.
- $x - y = 30 \pm 2$ m

When multiplying or dividing, add the **relative** uncertainties.
- $xy = (50$ m $\pm 2\%) (20$ m $\pm 5\%) = (1000$ m$^2 \pm 7\%) = (1000 \pm 70$ m$^2)$
- $x / t = (50$ m $\pm 2\%) / (25$ s $\pm 2\%) = (2$ m/s $\pm 4\%) = (2 \pm .08$ m/s$)$

With powers, scale the relative uncertainty by the power. (Square root is the one-half power.)
- $t^3 = (25$ s $\pm 2\%)^3 = (15625$ s$^3 \pm 6\%) = (15625 \pm 937$ s$^3) \approx (15600 \pm 900$ s$^3)$
- $\sqrt{(xy)} = \sqrt{(1000}$ m$^2 \pm 7\%) = (31.62$ m $\pm 3.5\%) = (31.62 \pm 1.1$ m$) \approx (31.6 \pm 1$ m$)$

Remember that when you add or scale a relative uncertainty, the result is still a relative uncertainty.

| Description | Operations |
|---|---|
| Numerical Uncertainty (Same units as value) | $\text{Quantity} = \text{Val} \pm \text{Unc}$ |
| Relative Uncertainty (Numerically dimensionless, but written as %, ppm, or ppb) | $\text{Quantity} = \text{Val} \pm \%\text{Unc}$ $\text{RelUnc} = \dfrac{\text{Unc}}{\text{Val}} \text{ or } \dfrac{\%\text{Unc}}{100\%} = \dfrac{\text{Unc}}{\text{Val}}$ |
| Addition and Subtraction: Add Numerical Unc | $\text{Value} = x_1 + x_2 + ...$ $\text{Unc} = \text{Unc}_1 + \text{Unc}_2 + ...$ |
| Multiplication and Division: Add %Unc (or RelUnc) | $\text{Value} = x_1 x_2$ $\%\text{Unc} = \%\text{Unc}_1 + \%\text{Unc}_2 + ...$ |
| Powers and Roots: Scale %Unc (or RelUnc) | $\text{Value} = x_1^{\text{Power1}}/x_2^{\text{Power2}}$ $\%\text{Unc} = \%\text{Unc}_1 \times \text{Power}_1 + \%\text{Unc}_2 \times \text{Power}_2 + ...$ |

# Alternate Method:  Max/Min Method

An alternative way of calculating the uncertainty of a result is to try to find the maximum and minimum possible values of the result, given that the input values may vary by their uncertainties.  As an example calculation, we know that the displacement of an accelerated object that starts from rest is related to the time:  $\Delta x = \frac{1}{2} at^2$.  Solving for the acceleration, we get $a = \boldsymbol{2\Delta x/ t^2}$


Note that the maximum $a$ comes from considering the maximum $\Delta x$ and the minimum $t$ (because $t$ is in the denominator).  You can often get away with just calculating the maximum, because the minimum is about as far below the average as the maximum is above the average.

Here is a sample calculation with the max/min method:

|  | Value | Max result | Min result |
|---|---|---|---|
| Δx | 3.0 m ± 0.05 m | 3.05 m | 2.95 m |
| t | 1.2 s ± 0.02 s | 1.18 s | 1.22 s |
| Calculations of $a = \frac{2\,\Delta x}{t^2}$ | 4.17 m/s² | 4.38 m/s² | 3.96 m/s² |
| Error in value | | 0.21 m/s² | 0.21 m/s² |
| $a$ with uncertainty | 4.17 ± 0.21 m/s² | | |